

Effect of viscous dissipation and radiation on natural convection in a porous medium embedded within vertical annulus

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Abstract

The effect of viscous dissipation and thermal radiation on natural convection in a porous medium embedded within a vertical annular cylinder is investigated. The inner surface of the cylinder is maintained at an isothermal temperature T_w and the outer surface is maintained at ambient temperature T_∞ . The fluid is assumed to obey the Darcy law. Finite element method is used to solve the partial differential equations governing the fluid flow and heat transfer behavior. The study is focused to investigate the combined effect of viscous dissipation and radiation. Results are presented for different values of the viscous dissipation parameter, radiation parameter, radius ratio, aspect ratio and Rayleigh number. It is observed that the viscous dissipation parameter reduces the average Nusselt number at hot surface. However, the average Nusselt number increases at the cold surface due to increased viscous dissipation parameter.

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1. Introduction

It is well known fact that the interest in the study of porous media has increased considerably in the recent years. This can be attributed to the many applications wherein porous medium is evolved such as thermal insulation of buildings, energy recovery of petroleum resources, solar energy collectors, chemical reactors, and nuclear waste disposals to name but few. A good insight of the subject is given in the recent books by Nield and Bejan [1], Ingham and Pop [2], Vafai [3], Pop and Ingham [4]. There is abundant literature available on different aspects of the porous media but still there are some aspects that have attracted little attention. The effect of viscous dissipation is one such area. Fand and Buckner [5], and Fand et al. [6] have studied the viscous dissipation effect on natural convection in horizontal cylinder embedded in the porous medium. Their study showed that the viscous dissipation might not be neglected. Recently Saeid and Pop [7] have studied the viscous dissipation effect

on natural convection in a porous cavity and found that the heat transfer rate at hot surface decreases with increase of viscous dissipation parameter. Israel-Cookey et al. [8] have studied the effect of viscous dissipation and radiation on unsteady magneto-hydrodynamic free-convection flow past vertical plate in a porous medium. They found that the temperature profile increases when viscous dissipation increases. Yih [9] has studied the viscous and Joule heating effects on non-Darcy MHD natural convection flow over an isoflux permeable sphere in porous medium. El-Amin [10] considered combined effect of viscous dissipation and Joule heating on MHD forced convection over a non-isothermal horizontal cylinder embedded in a fluid saturated porous medium. El-Amin [11] also studied the effect of magnetic field and viscous dissipation on power law fluids on a plate embedded in the porous medium. Yih [12] has studied the effect of radiation on natural convection over a vertical cylinder using finite difference method. Hossain and Alim [13] have investigated the natural convection radiation interaction on the boundary layer flow along vertical cylinder by using two different methods i.e. local non-similarity method and implicit finite difference scheme with Keller box elimination method. The effect of radiation on Darcy free convection flow along an

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Nomenclature

A_r	aspect ratio
c_p	specific heat $\text{J kg}^{-1} \text{K}^{-1}$
g	gravitational acceleration m s^{-2}
H	height of cylinder m
k	thermal conductivity $\text{W m}^{-1} \text{K}^{-1}$
K	permeability of porous media m^{-2}
L	$= r_0 - r_i$ m
\overline{Nu}	average Nusselt number
q_r	radiation flux W m^{-2}
r, z	cylindrical co-ordinates m
\bar{r}, \bar{z}	non-dimensional co-ordinates
R_r	radius ratio
Ra	Rayleigh number
R_d	radiation parameter
T, \bar{T}	dimensional and non-dimensional temperature respectively K
u, w	velocity in r and z directions m s^{-1}

Greek symbols

α	thermal diffusivity $\text{m}^2 \text{s}^{-1}$
β_T	coefficient of thermal expansion 1K^{-1}
β_R	absorption coefficient 1m^{-1}
ε	viscous dissipation parameter
ρ	density kg m^{-3}
ν	coefficient of kinematic viscosity $\text{m}^2 \text{s}^{-1}$
σ	Stephan Boltzmann constant $\text{W m}^{-2} \text{K}^{-4}$
ψ	stream function $\text{m}^{-3} \text{s}^{-1}$
$\bar{\psi}$	non-dimensional Stream function

Subscripts

h	hot
∞	conditions at outer radius
i	inner
o	outer
f	fluid

inclined surface in porous media was investigated by Hossain and Pop [14]. Raptis [15] has investigated the radiation and free convection flow in a porous medium with suction velocity. The porous medium embedded in vertical annular cylinder is an important area which finds engineering applications in heat exchangers, insulated pipe lines, gas cooled reactor vessels, bio-mass converters etc., Rajamani et al. [16] have studied the natural convective heat transfer in an annular cylinder.

The present study is undertaken to investigate the combined effect of viscous dissipation and radiation on the heat transfer from a porous medium embedded within a vertical annular cylinder. To the best of our knowledge, effect of viscous dissipation and radiation on the heat transfer through vertical annulus filled with saturated porous medium has not been reported so far. In this study, finite element method has been used to solve the governing partial differential equations. Results are presented in terms of the Nusselt number, streamlines and isotherms for various parameters.

2. Analysis

An annular vertical cylinder of inner radius r_i and outer radius r_0 filled with the saturated porous medium is considered. The inner surface of annulus is maintained isothermally at T_w and the outer surface has isothermal temperature T_∞ such that $T_w > T_\infty$. The top and bottom surfaces of the annulus are adiabatic. The r and z -axis points towards the width and height of the porous medium respectively. Following assumptions are applied;

- Porous medium is saturated with fluid.
- The fluid is assumed to be gray emitting and absorbing but non-scattering.
- The fluid and medium are in local thermal equilibrium everywhere inside the medium.

(d) The porous medium is isotropic and homogeneous.

(e) Fluid properties are constant except the variation of density.

The governing equations for the problem under consideration are given as:

$$\frac{\partial(ru)}{\partial r} + \frac{\partial(rw)}{\partial z} = 0 \quad (1)$$

$$\frac{\partial w}{\partial r} - \frac{\partial u}{\partial z} = \frac{gK\beta}{\nu} \frac{\partial T}{\partial r} \quad (2)$$

$$u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \alpha \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right) - \frac{1}{\rho c_p} \frac{1}{r} \frac{\partial}{\partial r} (r q_r) + \frac{\mu}{K(\rho c_p)_f} (u^2 + w^2) \quad (3)$$

The continuity equation (1) can be satisfied by introducing the stream function ψ as:

$$u = -\frac{1}{r} \frac{\partial \psi}{\partial z}, \quad w = \frac{1}{r} \frac{\partial \psi}{\partial r} \quad (4)$$

The boundary conditions are:

$$\text{at } r = r_i, \quad T = T_w, \quad u = 0 \quad (5a)$$

$$\text{at } r = r_o, \quad T = T_\infty, \quad u = 0 \quad (5b)$$

$$\text{at } z = 0 \quad \text{and} \quad z = H, \quad \frac{\partial T}{\partial z} = 0 \quad (5c)$$

Invoking Rosseland approximation for radiation

$$q_r = -\frac{4\sigma}{3\beta_R} \frac{\partial T^4}{\partial r} \quad (6)$$

Expanding T^4 about T_∞ using Taylor series and neglecting higher order terms [15] results:

$$T^4 \approx 4TT_\infty^3 - 3T_\infty^4 \quad (7)$$

Table 1
 \overline{Nu} comparison with available literature

Aspect ratio	Rajamani et al. [16]	Nath and Satyamurthy [19]	Prasad and Kulacki [20]	Present
3	3.868	3.81	3.70	3.8838
5	3.025	3.03	3.00	3.0638
8	2.403	2.45	2.35	2.4249

The following non-dimensional variables are used:

$$\begin{aligned} \bar{r} &= \frac{r}{L}, & \bar{z} &= \frac{z}{L}, & \bar{\psi} &= \frac{\psi}{\alpha L}, & \bar{T} &= \frac{(T - T_\infty)}{(T_w - T_\infty)} \\ R_d &= \frac{4\sigma T_\infty^3}{\beta_R k}, & Ra &= \frac{g\beta_T \Delta T K L}{\nu \alpha} \\ \varepsilon &= \frac{\alpha \mu}{(T_w - T_\infty) K (\rho c_p)_f} \end{aligned} \tag{8}$$

After substituting the above non-dimensional parameters, Eqs. (2) and (3) take the form:

$$\frac{\partial^2 \bar{\psi}}{\partial \bar{z}^2} + \bar{r} \frac{\partial}{\partial \bar{r}} \left(\frac{1}{\bar{r}} \frac{\partial \bar{\psi}}{\partial \bar{r}} \right) = \bar{r} Ra \frac{\partial \bar{T}}{\partial \bar{r}} \tag{9}$$

$$\begin{aligned} &\frac{1}{\bar{r}} \left[\frac{\partial \bar{\psi}}{\partial \bar{r}} \frac{\partial \bar{T}}{\partial \bar{z}} - \frac{\partial \bar{\psi}}{\partial \bar{z}} \frac{\partial \bar{T}}{\partial \bar{r}} \right] \\ &= \left(\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\left(1 + \frac{4R_d}{3} \right) \bar{r} \frac{\partial \bar{T}}{\partial \bar{r}} \right) + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} \right) \\ &+ \varepsilon \left[\left(\frac{1}{\bar{r}} \frac{\partial \bar{\psi}}{\partial \bar{r}} \right)^2 + \left(\frac{1}{\bar{r}} \frac{\partial \bar{\psi}}{\partial \bar{z}} \right)^2 \right] \end{aligned} \tag{10}$$

The corresponding boundary conditions are

$$\text{at } r = \bar{r}_i, \quad \bar{T} = 1, \quad \bar{\psi} = 0 \tag{11a}$$

$$\text{at } r = \bar{r}_o, \quad \bar{T} = 0, \quad \bar{\psi} = 0 \tag{11b}$$

$$\text{at } \bar{z} = 0 \quad \text{and} \quad \bar{z} = A_r, \quad \frac{\partial \bar{T}}{\partial \bar{z}} = 0 \tag{11c}$$

3. Numerical method

Eqs. (9) and (10) are coupled partial differential equations to be solved in order to predict the heat transfer behavior. These equations are solved by using finite element method. A simple 3 noded triangular element is used. \bar{T} and $\bar{\psi}$ vary linearly inside the element and can be expressed as:

$$\bar{T} = N_1 \bar{T}_1 + N_2 \bar{T}_2 + N_3 \bar{T}_3 \tag{12a}$$

$$\bar{\psi} = N_1 \bar{\psi}_1 + N_2 \bar{\psi}_2 + N_3 \bar{\psi}_3 \tag{12b}$$

where N_1, N_2, N_3 are shape functions.

Integrating Eqs. (9) and (10) using Galerkin method yields two coupled matrix form of equations. Details of FEM formulations can be obtained from [17,18]. These coupled matrix equations for an element are assembled to get the global matrix equation for the whole domain, which is solved iteratively to obtain \bar{T} and $\bar{\psi}$ in the porous medium. In order to get accurate results, tolerance level of solution for \bar{T} and $\bar{\psi}$ are set at 10^{-5} and 10^{-9} respectively. The tolerance level indicates the difference between previous iteration and current iteration for

a variable at all nodes. Fine mesh is used near the surfaces due to large variations in \bar{T} and $\bar{\psi}$ in these areas. The domain is meshed in such a way that the size of elements near the surface is 10 times smaller than those elements placed at the center of the domain. Sufficiently dense mesh is chosen to make the solution mesh invariant. The grid refining tests are carried out before selecting a mesh size of 1800 elements. It was found that the Nusselt number varied negligibly when the mesh size was increased from 1800 elements to 5000 elements. However the time required for 5000 elements was found to be 23.5 times greater than that of 1800 elements. The results are presented in terms of average Nusselt number at the hot as well as cold surfaces of the annulus with respect to various parameters such as the aspect ratio (H/L), radius ratio $(r_o - r_i)/r_i$, viscous dissipation parameter, radiation parameter and Rayleigh number.

The Nusselt number may be defined as:

$$h(T_w - T_\infty) = - \left[\left(\frac{16\sigma T_\infty^3}{3\beta_R} + k \right) \frac{\partial T}{\partial r} \right]_{r=r_i, r_o} \tag{13a}$$

$$Nu = \frac{hL}{k} \tag{13b}$$

With the help of Eqs. (13a) and (13b), the average Nusselt number can be evaluated as.

$$\overline{Nu} = - \frac{\int_0^{A_r} \left(1 + \frac{4R_d}{3} \right) \frac{\partial \bar{T}}{\partial \bar{r}} \Big|_{\bar{r}=\bar{r}_i, \bar{r}_o}}{A_r} \tag{13c}$$

4. Results and discussion

In order to verify the present methodology, results are compared with the available literature [16,19,20]. The comparison is shown in Table 1, which corresponds to the values $R_r = 1$, $Ra = 100$, $R_d = 0$, $\varepsilon = 0$. It is clear from this table that the present method is accurate enough to predict the heat transfer from the porous medium embedded in vertical annulus. Fig. 1 shows the variation of average Nusselt number at hot as well as cold surfaces with respect to aspect ratio of the annulus. This figure is obtained for the values $Ra = 100$, $R_r = 1$, $R_d = 1$. As expected, the average Nusselt number initially increases, reaches to maximum value and then starts declining when aspect ratio is increased. It can be seen that the effect of viscous dissipation parameter is to reduce the \overline{Nu} at hot surface. The effect of ε is higher at higher aspect ratio than at lower aspect ratio. The maximum value of \overline{Nu} shifts slightly to lower aspect ratio with increased ε . At $A_r = 1$, the \overline{Nu} decreased by 15.2% when ε is increased from 0 to 0.005. The corresponding decrease at $A_r = 10$ is found to be 88.9%. It is seen that the decrease in \overline{Nu} beyond $A_r = 1$ becomes sharp and then gradual, at high value of ε . In contrast to hot surface, the \overline{Nu} at cold surface increases with the increased value of ε . At cold surface also the \overline{Nu} initially increases, reaches maximum value and then declines. At higher value of viscous dissipation parameter, the decrease in \overline{Nu} with respect to A_r is very small after reaching the maximum value. The maxima shift towards higher A_r at higher value of ε .

Fig. 2 illustrates the effect of radius ratio and viscous dissipation parameter on the average Nusselt number. This figure

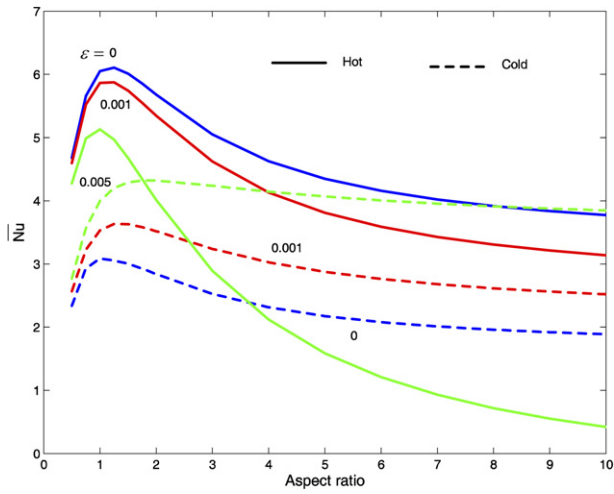


Fig. 1. \overline{Nu} variations with aspect ratio of annulus.

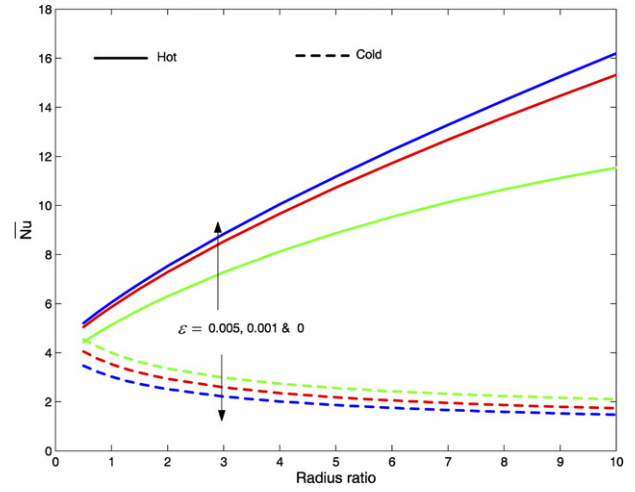


Fig. 2. \overline{Nu} variations with radius ratio of annulus.

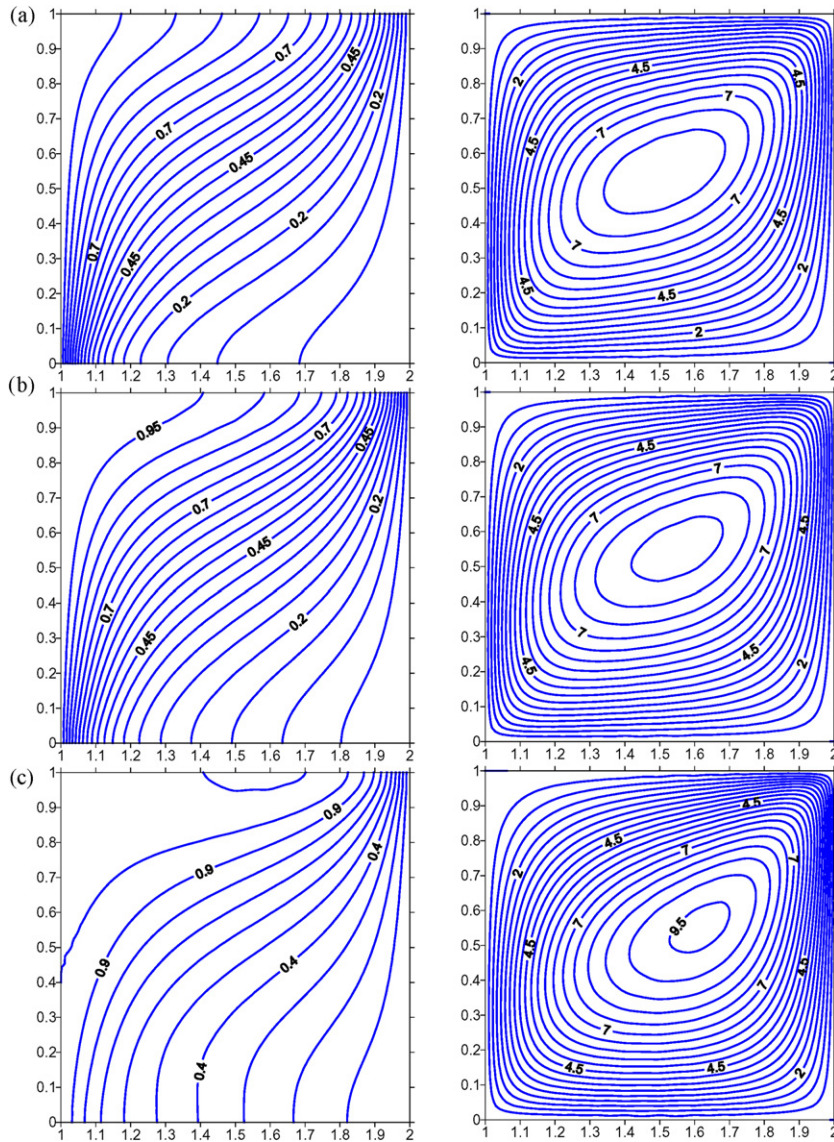


Fig. 3. Isotherms (left) and streamlines (right) for $Ra = 100$, $R_d = 1$, $A_r = 1$, $R_r = 1$: (a) $\epsilon = 0$; (b) $\epsilon = 0.01$; (c) $\epsilon = 0.03$.

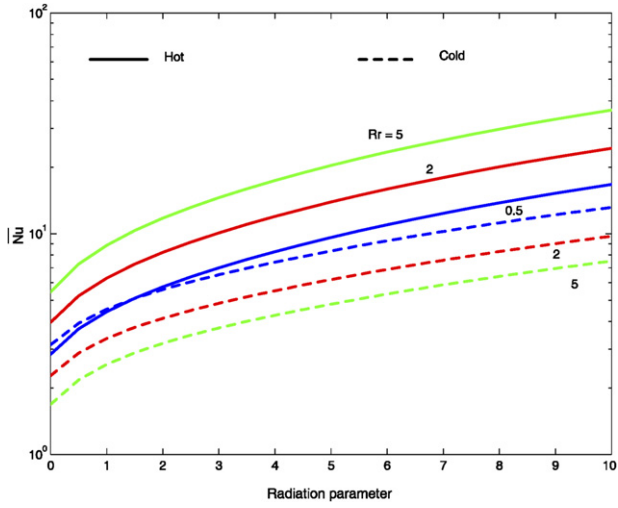


Fig. 4. \overline{Nu} variations with radiation parameter number.

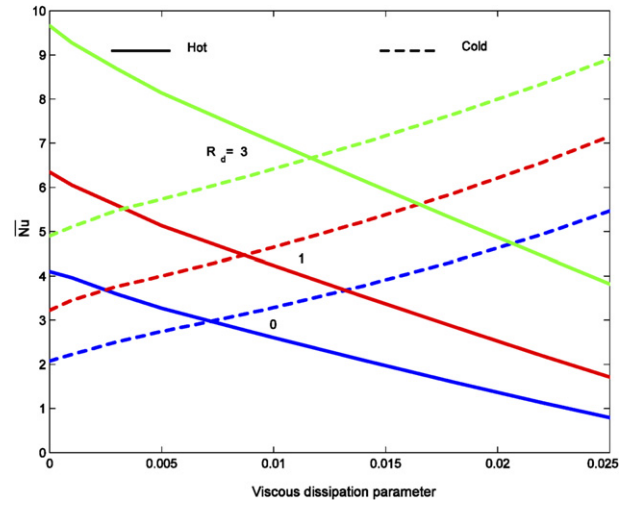


Fig. 5. \overline{Nu} variations with viscous dissipation parameter.

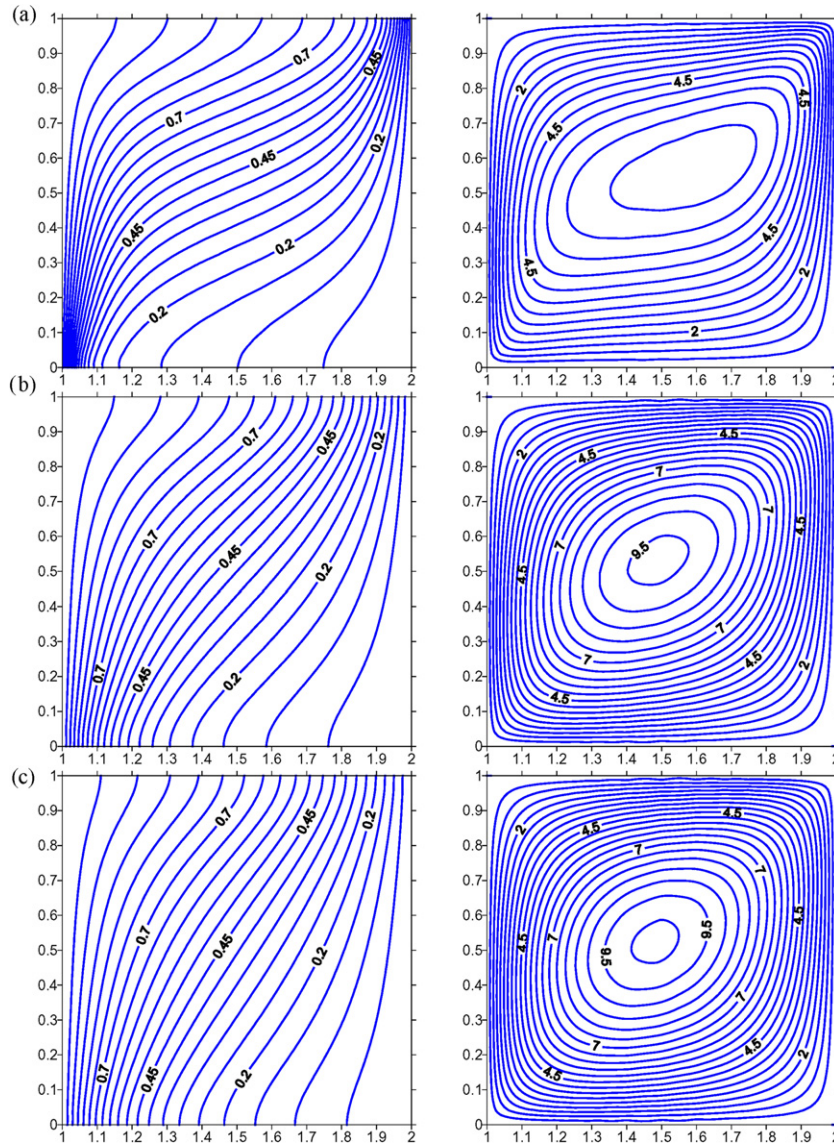


Fig. 6. Isotherms (left) and streamlines (right) for $Ra = 100$, $\varepsilon = 0.001$, $Ar = 1$, $Rr = 1$: (a) $R_d = 0$; (b) $R_d = 3$; (c) $R_d = 5$.

corresponds to the values $Ra = 100$, $A_r = 1$, $R_d = 1$. The \overline{Nu} at hot surface increases with increase in radius ratio. However \overline{Nu} decreases with increase in radius ratio at the cold surface of the annulus. Here also it can be seen that the effect of viscous dissipation parameter is to reduce the \overline{Nu} at hot surface, and increase at cold surface. However it is seen that the effect of ε is more at hot surface than at cold surface. The reduction in \overline{Nu} for hot surface is more pronounced at higher radius ratio when ε increases.

Fig. 3 shows the streamlines and isotherms inside the porous medium for various values of the viscous dissipation parameter. It can be seen that the isothermal lines move away from the hot surface when viscous dissipation parameter is increased. This indicates that the temperature difference near the hot surface decreases with increase in ε . This happens due to the reason that the viscous dissipation leads to local heat generation, which increases the temperature in the porous medium. Since the temperature of hot surface T_w is constant, the increased temperature of the porous medium reduces the temperature difference between the hot surface and nearby region. Due to this reason the heat transfer from hot surface to the porous medium decreases which results in decreased Nusselt number as shown in Figs. 1, 2. Contrary to hot surface, the isothermal lines move towards the cold surface due to increase in ε . Since the cold surface has lower temperature than the porous medium, the local heat generation in the porous medium increases the temperature difference between the cold surface and the nearby region in porous medium. This leads to increase in Nusselt number at cold surface. It is obvious from streamlines that the fluid movement shifts towards the cold surface due to increase in ε . Fig. 4 shows the variation of average Nusselt number with respect to radiation parameter. The \overline{Nu} increases at hot as well as cold surface due to increase in the R_d . As explained earlier, the \overline{Nu} increases at hot surface due to increase in radius ratio and decrease at cold surface. The difference in Nusselt number at hot and cold surfaces increases with the increase in radius ratio.

Fig. 5 demonstrates the effect of viscous dissipation and radiation parameter on average Nusselt number. This figure is obtained for $Ra = 100$, $A_r = 1$, $R_r = 1$. The \overline{Nu} at hot surface always decreases with increase in viscous dissipation parameter. The decrease and increase of Nusselt number at hot and cold surfaces respectively becomes steeper when radiation parameter is increased. The average Nusselt number at hot surface increased by 4.30 times when the radiation parameter is varied from 0 to 3 at $\varepsilon = 0.025$. The corresponding increase in the Nusselt number at cold surface is found to be 0.61 times at $\varepsilon = 0.025$. Fig. 6 illustrates the isotherms and streamlines distribution inside the porous medium due to varying radiation parameter. The isothermal lines tend to be parallel with the vertical surface due to increased radiation parameter and streamlines smoothly follow the geometry of annulus. This indicates that the conduction effect increases due to increased radiation parameter.

Fig. 7 shows the effect of Rayleigh number on the average Nusselt number. This figure corresponds to $R_d = 5$, $A_r = 1$, $R_r = 1$. When there is no viscous dissipation then the \overline{Nu} at hot surface always increase with increase in Rayleigh number. At

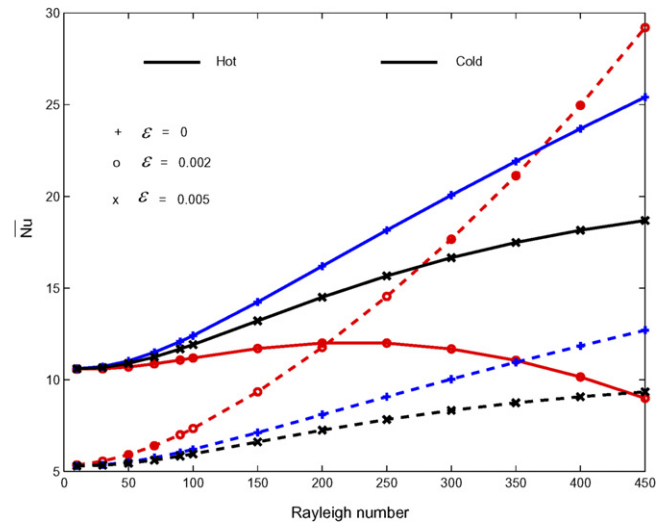


Fig. 7. \overline{Nu} variations with Rayleigh number.

$\varepsilon = 0.005$ the average Nusselt number increases slightly until $Ra = 200$ and then starts declining. This happens due to reason that higher Rayleigh number leads to high buoyancy force and thus faster fluid movement. Faster fluid movement enhances the friction between the fluid and solid matrix thus increasing the local heat generation, which in turn reduces the Nusselt number. \overline{Nu} at the cold surface always increases with increase in Rayleigh number.

5. Conclusion

The effect of viscous dissipation and radiation in a porous medium embedded in a vertical annulus is investigated using finite element method. The average Nusselt number at hot surface decreases with the increase in viscous dissipation parameter. However viscous dissipation parameter leads to increased Nusselt number at cold surface of the annulus. Beyond $A_r = 1$, the decrease in \overline{Nu} at hot surface is sharp at higher values of viscous dissipation parameter. However when ε is increased, the decrease in \overline{Nu} with respect to A_r is very small after reaching maximum value. \overline{Nu} at hot surface increases due to increase in the radius ratio but it decreases at cold surface when radius ratio is increased. Radiation parameter leads to increased \overline{Nu} at the hot as well as cold surfaces of the annulus.

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